

Modules close to SSP- and SIP-modules

Abyzov A., Nhan T., Quynh T.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© 2017, Pleiades Publishing, Ltd. In this paper, we investigate some properties of SIP, SSP and CS-Rickart modules. We give equivalent conditions for SIP and SSP modules; establish connections between the class of semisimple artinian rings and the class of SIP rings. It shows that R is a semisimple artinian ring if and only if RR is SIP and every right R -module has a SIP-cover. We also prove that R is a semiregular ring and $J(R) = Z(RR)$ if only if every finitely generated projective module is a CS-Rickart module which is also a C_2 module.

<http://dx.doi.org/10.1134/S1995080217010024>

Keywords

CS-Rickarts, SIP-CS, SIP-module, SSP-module

References

- [1] A. N. Abyzov and A. A. Tuganbaev, "Modules in which sums or intersections of two direct summands are direct summands," *Fundam. Prikl. Mat.* 19, 3–11 (2014).
- [2] A. N. Abyzov and T. H. N. Nhan, "CS-Rickart modules," *Lobachevskii J. Math.* 35, 317–326 (2014).
- [3] M. Alkan and A. Harmanci, "On summand sum and summand intersection property of modules," *Turk. J. Math.* 26, 131–147 (2002).
- [4] I. Amin, Y. Ibrahim, and M. F. Yousif, "D3-modules," *Commun. Algebra* 42 (2), 578–592 (2014).
- [5] I. Amin, Y. Ibrahim, and M. F. Yousif, "C3-modules," *Algebra Colloq.* 22, 655–670 (2015).
- [6] F. W. Anderson and K. R. Fuller, *Rings and Categories of Modules* (Springer, New York, 1974).
- [7] V. Camillo, Y. Ibrahim, M. Yousif, and Y. Zhou, "Simple-direct-injective modules," *J. Algebra* 420, 39–53 (2014).
- [8] J. Clark, C. Lomp, N. Vanaja, and R. Wisbauer, *Lifting Modules. Supplements and Projectivity in Module Theory*, *Frontiers in Mathematics* (Birkhäuser, Basel, Boston, Berlin, 2006).
- [9] N. V. Dung, D. V. Huynh, P. F. Smith, and R. Wisbauer, *Extending Modules*, Vol. 313 of *Pitman Research Notes in Mathematics* (Longman Sci. and Tech., Harlow, New York, 1994).
- [10] E. E. Enochs, "Injective and flat covers, envelopes and resolvents," *Israel J. Math.* 39, 189–209 (1981).
- [11] C. Faith, *Algebra II. Ring Theory* (Springer, New York, 1967).
- [12] J. L. Garcia, "Properties of direct summands of modules," *Commun. Algebra* 17, 73–92 (1989).
- [13] A. Hamdouni, A. Harmanci, and A. Ç. Özcan, "Characterization of modules and rings by the summand intersection property and the summand sum property," *JP J. Algebra, Number Theory Appl.* 5, 469–490 (2005).
- [14] J. Hausen, "Modules with the summand intersection property," *Commun. Algebra* 17, 135–148 (1989).
- [15] Y. Ibrahim, M. Tamer Kossan, T. C. Quynh, and M. Yousif, "Simple-direct-projective modules," *Comm. Algebra* 44, 5163–5178 (2016).
- [16] G. Lee, S. T. Rizvi, and C. S. Roman, "Rickart modules," *Commun. Algebra* 38, 4005–4027 (2010).
- [17] G. Lee, S. T. Rizvi, and C. S. Roman, "Direct sums of Rickart modules," *J. Algebra* 353, 62–78 (2012).

- [18] S. H. Mohammed and B. J. Müller, Continuous and Discrete Modules, Vol. 147 of London Math. Soc. Lecture Notes Series (Cambridge Univ. Press, Cambridge, 1990).
- [19] W. K. Nicholson, "Semiregular modules and rings," *Canad. J. Math.* 28, 1105-1120 (1976).
- [20] W. K. Nicholson and M. F. Yousif, *Quasi-Frobenius Rings* (Cambridge Univ. Press, Cambridge, 2003).
- [21] T. C. Quynh, M. T. Kusan, and L. V. Thuyet, "On (semi)regular morphisms," *Commun. Algebra* 41, 2933-2947 (2013).
- [22] G. V. Wilson, "Modules with the summand intersection property," *Commun. Algebra* 14, 21-38 (1986).
- [23] R. Wisbauer, *Foundations of Module and Ring Theory* (Gordon and Breach, London, 1991).